Finite element approach for simulation of gravity instability in the felsic crust and lithosphere mantle

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A new approach is proposed in this work to outline the processes of partial melting and gravity instability in lithospheric mantle and crust. The model was constructed to describe the melting and related upwelling of felsic material caused by underplating of basic magma at the base of continental crust (see modeling results in Figure 2) and to describe the interaction of a hot mantle plume with shield lithosphere (see modeling results in Figure 3).

Setting up the problem is shown in Figure 1. A two-dimensional rectangular area of the Earth's crust 45 × 800 km and lithospheric mantle 110 × 800 km in size (depth/width) is considered.

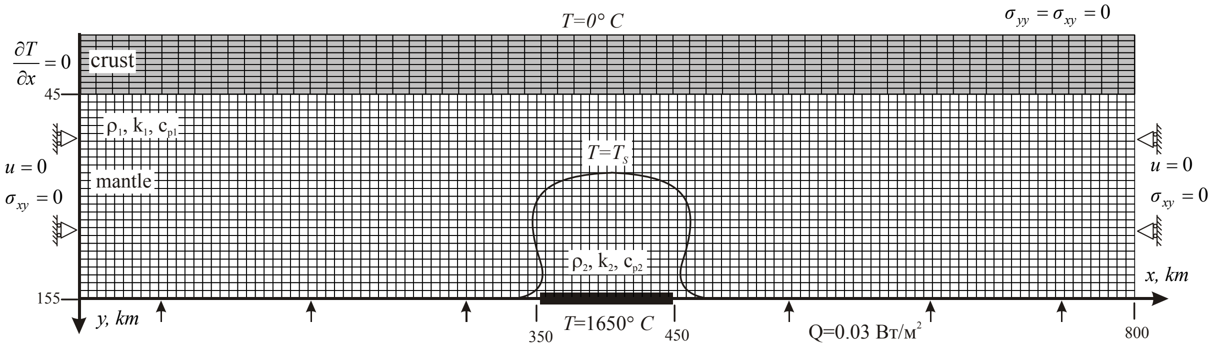

Figure 1. Setting up the problem of melting and diapirism in lithospheric mantle. The geometry and boundary conditions are shown.

The equations of mechanical equilibrium in "weak" form (the equation for the principle of the virtual velocities or equation for virtual power balance) (Korobeynikov et al., 2009) and the equation of thermal conductivity with variable coefficient and allowance for thermal effect of phase transition during melting are solved. The mathematic modeling of quasistatic deformation is based on equations that were solved numerically in terms of plain strain approximation of the problem of flat deformation. The finite element method was used for discretization of solid mechanics (SM) equations, which was applied by us earlier for modeling subduction, thrusting, and shears leading to large deformations (Polyansky et al., 2009; Babichev et al., 2009). The numerical modeling was performed using MSC.Marc software (Marc…, 2005), which involves all types of non-linearity of SM equations. Due to large deformations, a grid in the upper part was remeshed under the conditions of attainment of critical deformation of elements: change of angle between element edges more than 20° at average edge length of 300 m.

The ability for upwelling of light fraction in the surrounding medium is based on the Archimedean law. At least two conditions are required "to launch" the Archimedean law: the mass density of buoyant fraction should be less than the density of surrounding medium and the latter should be in liquid state (i.e. the stress tensor should have spherical shape). Melted phase is typically modeled on the basis of viscous liquid. The incompressible viscous Newtonian liquid is defined by the following equation *(Marc...,* 2005)

$$s_{ij} = 2\mu d_{ij}, \quad \sigma_{ij} = s_{ij} - p\delta_{ij}, \quad d_{kk} = 0. \tag{1}$$

Hereinafter s_{ij} are the components of deviator of the Cauchy stress tensor (whose components are noted by σ_{ij})

$$s_{ij} \equiv \sigma_{ij} - \sigma_m \delta_{ij}, \;\; \sigma_m \equiv \frac{1}{3}\sigma_{kk} = -p, \tag{2}$$

where δ_{ij} is the Kronecker delta, p is the pressure in liquid, μ is the viscosity. Hereinafter, the indices of components of tensors and vectors run across values 1, 2, 3; summing is conducted over repetitive indices from 1 to 3. In equations (1), the quantities d_{ij} designate the components of strain rate tensor, which together with components of vorticity tensor ω_{ij} represent the symmetrical and skew-symmetrical components of the velocity gradient tensor:

$$d_{ij} \equiv \frac{1}{2}(v_{i,j} + v_{j,i}), \;\; w_{ij} \equiv \frac{1}{2}(v_{i,j} - v_{j,i}). \tag{3}$$

where v_i are the components of the velocity vector, coma denotes the partial derivative of corresponding value with respect to spatial variable x_i.

In this work, the liquid phase is formulated by constitutive relations for non-elastic deformation of material, which is described for large deformations as relation between components of the Hill objective rates (Hill, 1958; Korobeynikov, 2000) of the Cauchy stress tensor and components of strain rate tensor in the following form:

$$\sigma_m^H = \frac{E}{1-2\nu}(d_m - d^{th}), \;\; s_{ij}^H = \frac{E}{1+\nu}(d'_{ij} - d_{ij}^n), \tag{4}$$

Where

$$\begin{aligned} &\sigma_m^H \equiv \frac{1}{3}\sigma_{ii}^H, \;\; s_{ij}^H \equiv \sigma_{ij}^H - \sigma_m^H \delta_{ij}, \;\; d^{th} = \alpha \dot{T}, \\ &d_m \equiv \frac{1}{3} d_{kk}, \;\; d'_{ij} \equiv d_{ij} - d_m \delta_{ij}, \end{aligned} \tag{5}$$

where d'_{ij}, and s_{ij}^H are components of strain rate tensor deviators and the Hill rate of the Cauchy strain tensor, respectively; d^{th} is the rate of temperature deformation; α is the coefficient of temperature expansion, T is the current temperature; hereinafter, the dot above variable denotes its material derivative.

The values d_{ij}^n denote the components of non-elastic deformation rate tensor and are determined below. The components of the Hill rate of the Cauchy stress tensor are formulated in the Cartesian coordinate system as follows (Korobeynikov, 2000)

$$\sigma_{ij}^H \equiv \dot{\sigma}_{ij} + s_{kj} w_{ki} + s_{ik} w_{kj} + s_{ij} d_{kk}. \tag{6}$$

Equations (4)–(6) define the deformation of compressible medium, taking into account only compressibility for elastic component. Condition of incompressibility is proposed for non-elastic constituent of strain rate tensor, which implies that the mean value of the strain rate of media d_m is related via elastic deformation law with the average rate of stress changes σ_m^H in (4), while sum d d_{kk} equals 0).

In order to describe non-elastic deformation of the material, we applied the law of proportionality of rate of non-elastic deformation tensor and deviator of the Cauchy stress tensor

$$d_{ij}^n = \gamma s_{ij}, \tag{7}$$

which is confirmed by numerous experimental data.

It is seen that expressions (1) and (7) have similar structure. In particular, identifying the components d_{ij}^n with d_{ij} at $\gamma = 1/2\mu$ provides equivalence of (1) and (7). The term γ in (7) for thermoelastoplastic material and thermoelastic material taking into account creep deformation is determined by different ways.

Two variants of the model of non-elastic deformation of medium material are considered. For ideal elastoplastic material with Huber–Von Mises yield function (note that only use of the Huber–Von Mises yield surface provides the equation (7) for the rate of plastic constituent of the strain rate tensor), the parameter γ is determined as follows (Korobeynikov, 2000)

$$\gamma = \begin{cases} 0, \text{ if } \sigma_e < \sigma_y \text{ or } \sigma_e = \sigma_y \text{ and } s_{ij} d_{ij} \le 0, \\ \frac{3}{2} s_{ij} d_{ij} / \sigma_e^2, \text{ if } \sigma_e = \sigma_y \text{ and } s_{ij} d_{ij} > 0. \end{cases} \tag{8}$$

The second variant of the model was based on the non-linear viscous, temperature dependent rheological model of medium. The creep deformation is expressed by relation

$$\gamma = \frac{3}{2} A_0 \sigma_e^{n-1} e^{-\frac{H}{RT}}, \tag{9}$$

where A_0, n, H are constants of rheological law, R is the universal gas constant, T is the temperature. The effective stress σ_e introduced in (8) and (9) is written as follows

$$\sigma_e \equiv \sqrt{\frac{3}{2} s_{ij} s_{ij}}. \tag{10}$$

Principal difference of constitutive relations (7), (8), and (7), (9) is that the first pair does not depend on natural time, while the second, depends. The constants of rheological laws (8)–(9) are listed in Table.

Modeling results are shown in Figures 2, 3. The problem of Rayleigh-Taylor gravitational instability under uniform heating of crustal layer from below was considered with aim of comparison of numerical versus analytical solution. Underplated basic magma layer with initial temperature of 1200°C at the basement of model is considered as permanent heat source. Periodical structure of uppwelling plumes with wave length of 25 km is formed (Fig. 2). Model of the penetration of ultrabasic magma plume though the lithosphere mantle was considered in relation to the Siberian flood basalt formation. The results are presented in Fig. 3 as a temperature evolution patterns. We observed that the plume head rises up to middle part of lithospheric mantle in case of viscous rheology even with continuous heating.

Table of model parameters

Parameter, symbol (dimension)	Value
Density of crust/mantle; ultrabasic melt ρ_0, (kg/m^3)	2800/3430 3350
Poisson's ratio, ν	0.25
Young's modulus, E (Pa)	$5\cdot10^{10}$
Heat capacity, c_p (J/(kg·K))	1250
Heat conductivity, k, J/(m sec K)	2.0
Crustal radiogenic heat, A_0 W/m^3	$2.0\cdot10^{-6}$
Heat flow, Q_s, (mW/m^2)	30
Heat expantion, α, (K^{-1})	$1.0\cdot10^{-5}$
Elastoplastic rheology (Gerya, Burg 2007)	
Yield stress, σ_y ,(Pa)	$1\ 10^6 - 5\ 10^8$
Viscoelastic rheology (Chopra, Patterson, 1984)	
Preexponential constant of viscosity, A, (Pa^{-n} sec^{-1})	$5.5\ 10^{-25}$
Activation energy, H, (KJ/mol)	498
Exponent of power, n	4.48

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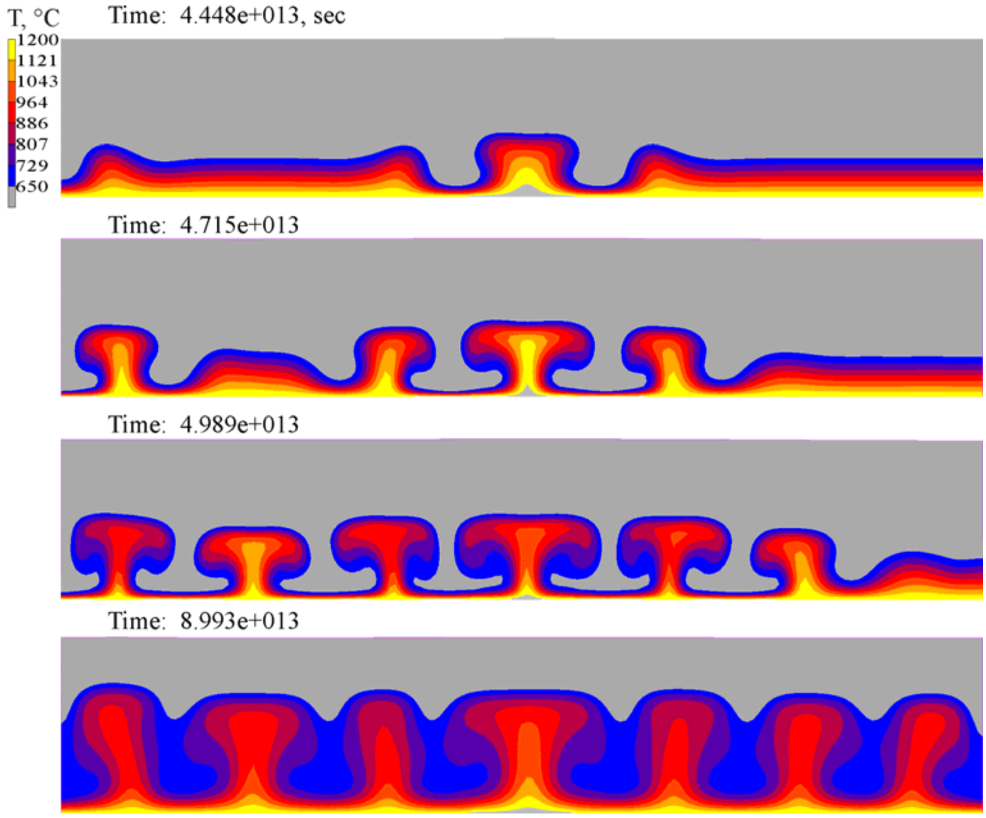

Figure 2. Temperature evolution during the upwelling of partial melted granite-gneiss material.

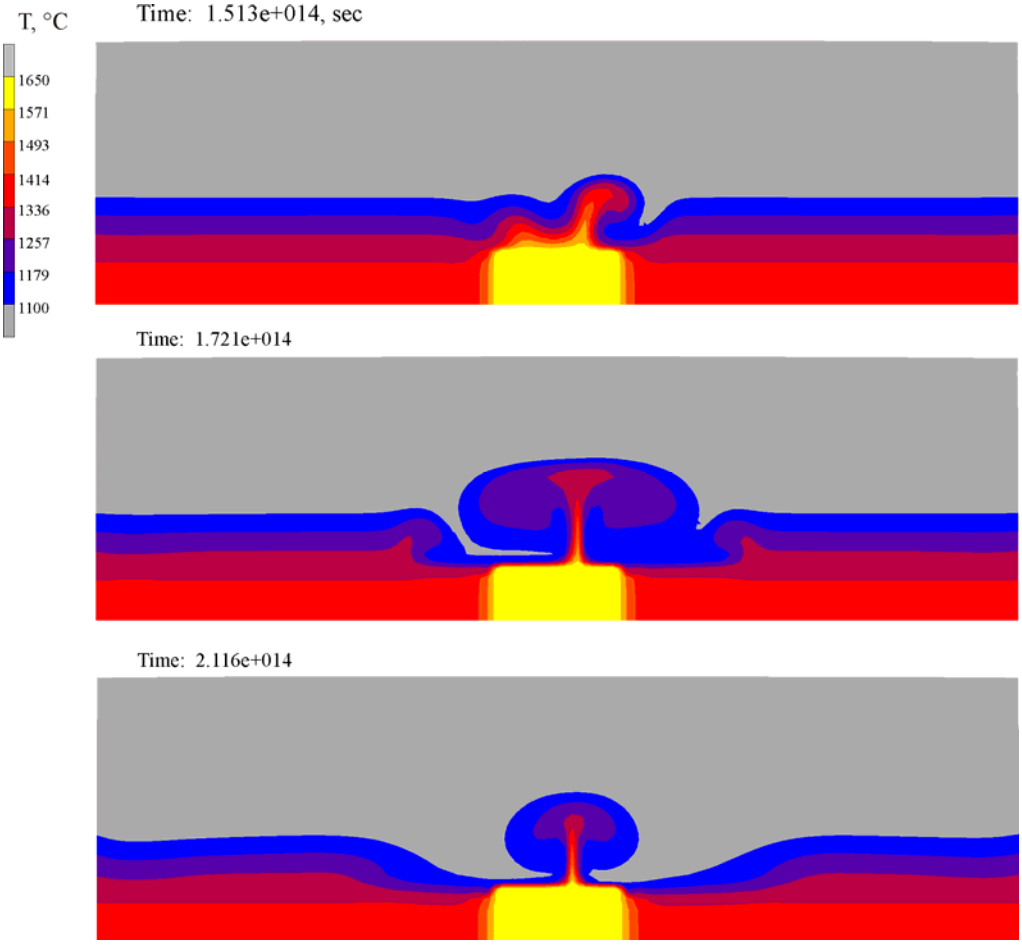

Figure 3. Temperature evolution during the upwelling of peridotite magma in the bottom of the lithosphere plate.